Comparison between approximate and exact measurement invariance on non-normal data

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Introduction

Measurement invariance

- an important assumption for multi-group comparisons

Multiple Group Confirmatory Factor Analysis (MG-CFA)

- MG-CFA established a series of models by constraining specific parameters to be equal across groups and tests them sequentially

$$X^g = \tau^g + \Lambda^g \xi^g + \delta^g$$
$$\Sigma^g = \Lambda^g \Phi^g \Lambda^{g'} + \Theta^g$$

- Configural Invariance: the factor structures are the same across groups
- Metric Invariance: $\Lambda^1 = \Lambda^2 = \cdots = \Lambda^G$
- Scalar Invariance: $\tau^1 = \tau^2 = \cdots = \tau^G$
- Error Variance Invariance: $\Theta^1 = \Theta^2 = \cdots = \Theta^G$
- Limitations: requires parameters to be exactly equal across groups, which would lead to poor model fit.

Bayesian Approximate Measurement Invariance

- imposes a normal prior distribution with a mean of zero and a very small variance on the cross-group parameter difference.
- The prior allows for a "wiggle room" around zero parameter difference

$$\Lambda^1 \approx \Lambda^2 \approx \cdots \approx \Lambda^G$$
$$\tau^1 \approx \tau^2 \approx \cdots \approx \tau^G$$

- Can detect parameters that strongly violating measurement invariance
- The approximate constraints is more reasonable and more flexible than strict exact constraints.

Non-normal distribution

- Common in applied studies

➢ About 80% psychometric data followed non-normal distribution

➢ Non-normal distribution can lead to biased maximum likelihood χ^2 and parameter estimation (Curran et al., 1996)

The present study

Monte Carlo simulation study

- Investigate and compare the performance of MG-CFA (exact measurement invariance, **EMI**) and Bayesian approximate measurement invariance (**AMI**) under non-normal data conditions.

Empirical study

- Analyzed a non-normal real dataset with EMI and AMI

Simulation Study

Methods

- Data generation model: single factor CFA model with 5-items
- Data generation conditions

① Number of groups: 3, 9, 15

② the first group was the reference group (mean = 0, SD = 1)

③ the factor means and variances of other groups were sampled from normal distributions N(0,0.3) and N(1,0.1), respectively (Pokropek et al., 2019)

④ Data distributions: Skewness = 0/1/3, excessive kurtosis= 0/7/21

⑤ Number of non-normal items： 1, 3, 5

⑥ Group sample sizes： 200, 500, 800

⑦ degree of non-invariance: normal distributions with variances: 0, 0.001, 0.005, 0.01, 0.05

Data analysis

- EMI： MG-CFA using MLR estimation
- AMI： Bayesian estimation using zero-mean priors with 5 variance: N(0, 0.001), N(0, 0.005), N(0, 0.01), N(0, 0.05)

Partial Results

- PPP results of AMI (N = 200; Figure 1): PPP decreased as non-normal items increased and as distribution deviate more from normal distribution.

Simulation Study

- RMSE for group mean difference estimation (skewness = 1, excessive kurtosis = 7; Figure 2): AMI and EMI performed similarly when number of groups was 3. RMSE of AMI was more prone to group number.

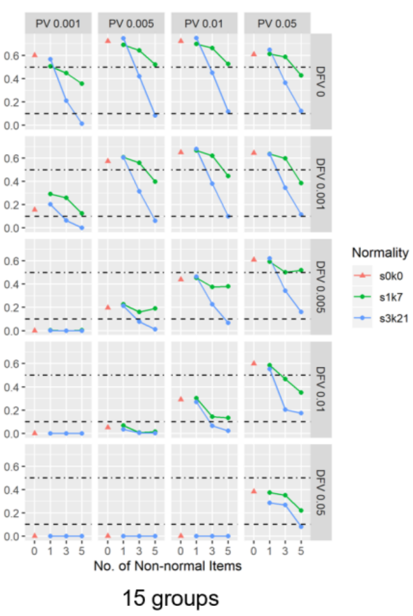

Figure 1. PPP results

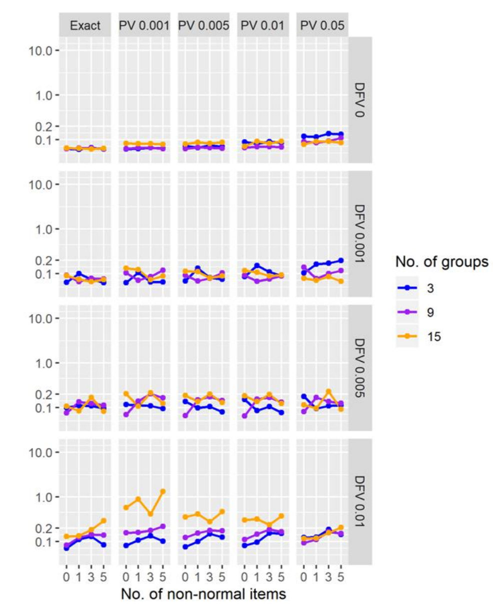

Figure 2. RMSE results

Note: Exact = EMI, PV = AMI with prior, s = skewness, k = excessive kurtosis, DFV = degree of non-invariance

Empirical Study

- An international study on Narcissism (Wetzel et al., 2020)
- Narcissistic Admiration and Rivalry Questionnaire (Back et al., 2013)
- Non-normal data (N=300) from 3 countries: British, Germany, Poland
- Group mean difference estimates (British as reference group)

➢ EMI: Germany - British: 0.007, p = 0.936; Poland - British: 0.998, p < 0.001

➢ AMI (0.05 prior variance): Germany - British： -0.104, p = 0.265; Poland – British: 0.894, p < 0.001

Discussion

- The influence of skewness, kurtosis, and number of non-normal items on PPP values could be explained by the computation of PPP
- Limitations of the current research

➢ Some simulation conditions were not ecological (e.g., group sample sizes were set to equal across groups)

➢ Fit indices of AMI such as PPP needs a deeper investigation.

Reference

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